**PSQF 7375 section 0006 Advanced Longitudinal Models HW1:   
Reviewing Concepts in Longitudinal Multilevel Models (10 points total)**

1. (ch. 1) How does a *between-person* relationship differ from a *within-person* relationship? Provide an example of each type from your own area of research or experience.  
     
   A between-person relationship compares one or more outcomes among people based on stable or constant attributes. A within-person relationship compares different measures of the same person over time. In longitudinal multilevel analysis, a between-person relationship is related to time-invariant predictors (level 2), whereas a within-person relationship is related to time-varying predictors (level 1). In my research area, education, we could have multiple grades for the same student throughout the academic year. A between-person relationship would consist of comparing the GPA for a specific year between males and females, whereas a within-person relationship would consist of analyzing the relationship between grades and time-varying predictors, for instance, any measures of motivation or self-efficacy that were collected during that academic year for each student.
2. (ch. 1 and 5) What is the difference between a *fixed effect* and a *random effect*? To which side of the model (means or variance) does each type of effect belong?  
     
   A fixed effect represents the constant contribution of a predictor to the estimate of an outcome and is the same for all individuals in the data set. In contrast, a random effect represents inter-individual variation around a fixed effect; in other words, the idea that each person needs his or her own version of a slope (random slope) or a starting point (random intercept). Fixed effects belong to the model for the mean and random effects to the model for the variance.
3. (ch. 3) Why is it necessary to consider what to include in the model for the variance in longitudinal data? Refer in your answer to the statistical and substantive reasons for doing so.

Statistically, it is important consider what to include in the model for the variance because having the right model for the variance will help to calculate the standard errors of the fixed effects of the model at level 1 (and thus their p-values). Furthermore, depending on the variance components (random effects and residual variance) we include, we have different alternatives for modeling the variance-covariance structure of the data (combining a G matrix with an R matrix to form V, or just an R matrix). Substantially, what we include in the model for variance will reflect the multiple sources of variance we have in our data set. If we analyze fluctuation, then we should explore different alternative covariance structures over time in the R matrix, or perhaps combine this matrix with a random intercept of the G matrix, since in longitudinal analyses the correlation between outcomes for the same person is common (and perhaps we can save parameters). Otherwise, if we have change over time, the model for the variance could have multiple random effects in the G matrix, which when combined with the R matrix (usually with constant variance and no covariance) can better fit the data.

1. (ch. 3) What kinds of model comparisons can be made using −2LL, AIC, and BIC indices when using maximum likelihood (ML) versus residual maximum likelihood (REML)? Provide examples.  
     
   In the case of AIC and BIC, we can use them to evaluate the relative fit of different non-nested models (different predictors or variance components) fitted with the same estimator (ML or REML). When we use regular ML, different nested models can be compared, either in the amount of fixed or random effects they have. However, when using REML, only nested models that differ in their variance components can be compared (thus the fixed effects between models must be the same).
2. (ch. 3) How does one fit the univariate and multivariate variants of repeated measures ANOVA models as multilevel models (i.e., what goes into each side of the model to do so)? What are the limitations of these ANOVA models *as traditionally estimated using ordinary least squares* for longitudinal data?  
     
   First, one must use the stacked version of the data, tricking the software to estimate a multilevel model. Then, depending on the variance components, a univariate or multivariate repeated measures model can be fitted using a compound symmetry or unstructured pattern for the residual variance matrix (R matrix), respectively (the model for variance is R only). It should be noted that in both models we can only have fixed effects, time is assumed to be balanced with equal intervals, and time must be included as a categorical variable (a saturated version of time in the model for the mean). Some limitations of these ANOVA models using OLS are, first, that cases are eliminated if any outcome is missing, second, that they assume a constant or unstructured pattern for the variance components, and third, that they require balanced time.
3. (ch. 4) What is the difference between alternative covariance structure models that use only the **R** matrix versus those that use the **G** and **R** matrices? What advantage does the latter have?  
     
   Alternative Covariance Structure (ACS) models using only the R matrix generally assume no variation between individuals over time. All variance is modeled using a variance-covariance pattern (independent, compound symmetry, AR or TOEP, and their heterogeneous versions) for the residual variance at level 1 (so the R matrix equals the total variance-covariance matrix). In contrast, ACS models using matrices G and R are a bit more flexible, since they include a source of constant between-person variance in matrix G (level 2), and model the remain residual variance in the R matrix with a specific pattern (level 1). Therefore, the new total matrix, the V matrix, might fit the data better. This is its main advantage, including the random intercept variance in the G matrix, which is a common source of dependency in longitudinal analyzes that when combined with other structures for the R matrix can help find a good model for the variance.
4. (ch. 5) What are the relationships among and the contents of the **Z**, **G**, **R**, and **V** matrices?   
     
   This information builds what is known as the ‘block diagonal’ structure, which represents the overall variance-covariance pattern used to model the per-person correlated outcomes that we have in the longitudinal analysis. The G matrix contains the random variance components of the model, while the Z matrix contains the values of the predictors with random effects in the model. The R matrix has the time-specific variance-covariance pattern of the remaining residual variance, and the V matrix is the diagonal block structure that results from putting these matrices together in the form of Vi = Zi \* Gi \* ZiT + Ri.
5. (ch. 5) How is dependency of residuals captured by adding random effects? Can adding random effects “explain” the variance in an outcome? Why or why not?  
     
   Adding random effects to the model we divide the outcome variance at level 1 into new level 2 variances for each source of person dependency found (random intercept and random slopes). By doing this, the residual variance becomes the time-specific deviation from the predicted outcome for the individual. Furthermore, after adding random effects, we assume that both levels of variance are independent and that the residual variance has constant variance with no covariance over time.

Although the residual variance of the outcome is reduced after adding random effects, random effects do not explain this reduction, but rather are new variance components that have been separated from the residual variance to quantify the variation of people around a fixed effect. Predictors at level 2 are those that can explain why people need their ‘own’ intercept or slopes in the model.

1. (ch. 6) What is the purpose of including fixed and random effects related to change over time? Which of these two types of effect should be included in the model before the other, and why?  
     
   The purpose is to have the best model to represent the effect of time on the result (the growth curve). Since the rest of the predictors at level 1 and level 2 will be added to explain the remaining residual variance or as moderators of the random intercept or random slopes, it is crucial to have the best model for time. In addition, different ways of modeling time should be tested (linear, quadratic, piecewise, exponential). In the end, the fixed effects describe an average behavior (or tendency), while the random effects capture the variability around this average, so they complement each other to analyze the effect of time on the result. To build a model, fixed effects must first be included, since random effects capture the variance around them.
2. (ch. 7) What are the roles of time-invariant predictors in the model for the means in a longitudinal analysis? How do these roles relate to which source of variance should be explained by each fixed effect of a time-invariant predictor?

Generally speaking, time-invariant predictors adjust the growth curve created by time (its fixed and random effects). These can have fixed and systematically varying effects. The fixed effects of the time-invariant predictors (main or simple) at level 2 adjust the random intercept and explain why people differ from each other constantly over time. They can also moderate the fixed effect of time slopes at level 1 in two circumstances. On the one hand, if the time slopes have random effects, the level 2 predictors can explain why people differ from that average effect (using cross-level interactions). On the other hand, if the time slope does not have a random effect, the level 2 predictors can explain part of the residual variance (systematically varying effects).